الجامعة السورية الخاصة
SYRIAN PRIVATE UNIVERSITY

كـلـيـــة هندسة الحاسوب والمعلوماتية والاتصالات
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Chapter_ 2

Number systems, operations and codes

Lecture _04

Hexadecimal and Octal Numbers

4-1. Hexadecimal Numbers

- Hexadecimal uses sixteen characters to represent numbers: the numbers 0 through 9 and the alphabetic characters A through F.
- The hexadecimal number system has a base of sixteen;

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Binary-to- Hexadecimal conversion

- Large binary number can easily be converted to hexadecimal by grouping 4 bits at a time and writing the equivalent hexadecimal character.

Example 3-14 Express 1001011000001110_2 in hexadecimal:

Solution Group the binary number by 4-bits starting from the right. Thus, 960E

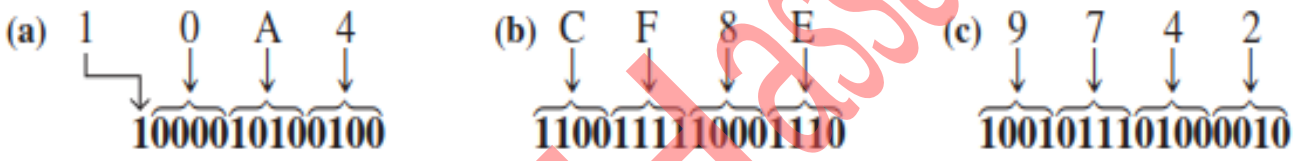
Hexadecimal-to- Binary conversion

- To convert from a hexadecimal number, revers process and replace each hexadecimal symbol with the appropriate bits.

Example 3-15 Determine the binary numbers for the following hexadecimal numbers:

(a) $10A4_{16}$ (b) $CF8E_{16}$ (c) 9742_{16}

Solution



In part (a), the MSD is understood to have three zeros preceding it, thus forming a 4-bit group.

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Hexadecimal-to- Decimal conversion

- **One way**: first convert the hexadecimal number to binary and then convert from binary to decimal.
- **Another way**: Hexadecimal is a weighted number system. The column weights are powers of 16, which increase from right to left.

Decimal	Hexadecimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Column weights $\begin{matrix} 16^3 & 16^2 & 16^1 & 16^0. \\ 4096 & 256 & 16 & 1\ . \end{matrix}$

Example 3-16

Solutions

1) Convert the following hexadecimal number to decimal: (a) $1C_{16}$ (b) $A85_{16}$

(a) 1 C

$\overbrace{0001}\overbrace{1100} = 2^4 + 2^3 + 2^2 = 16 + 8 + 4 = \mathbf{28}_{10}$

(b) A 8 5

$\overbrace{1010}\overbrace{1000}\overbrace{0101} = 2^{11} + 2^9 + 2^7 + 2^2 + 2^0 = 2048 + 512 + 128 + 4 + 1 = \mathbf{2693}_{10}$

2) Express $\mathbf{1A2F}_{16}$ in decimal.

Start by writing the column weights:

4096 256 16 1

1 A 2 F_{16}

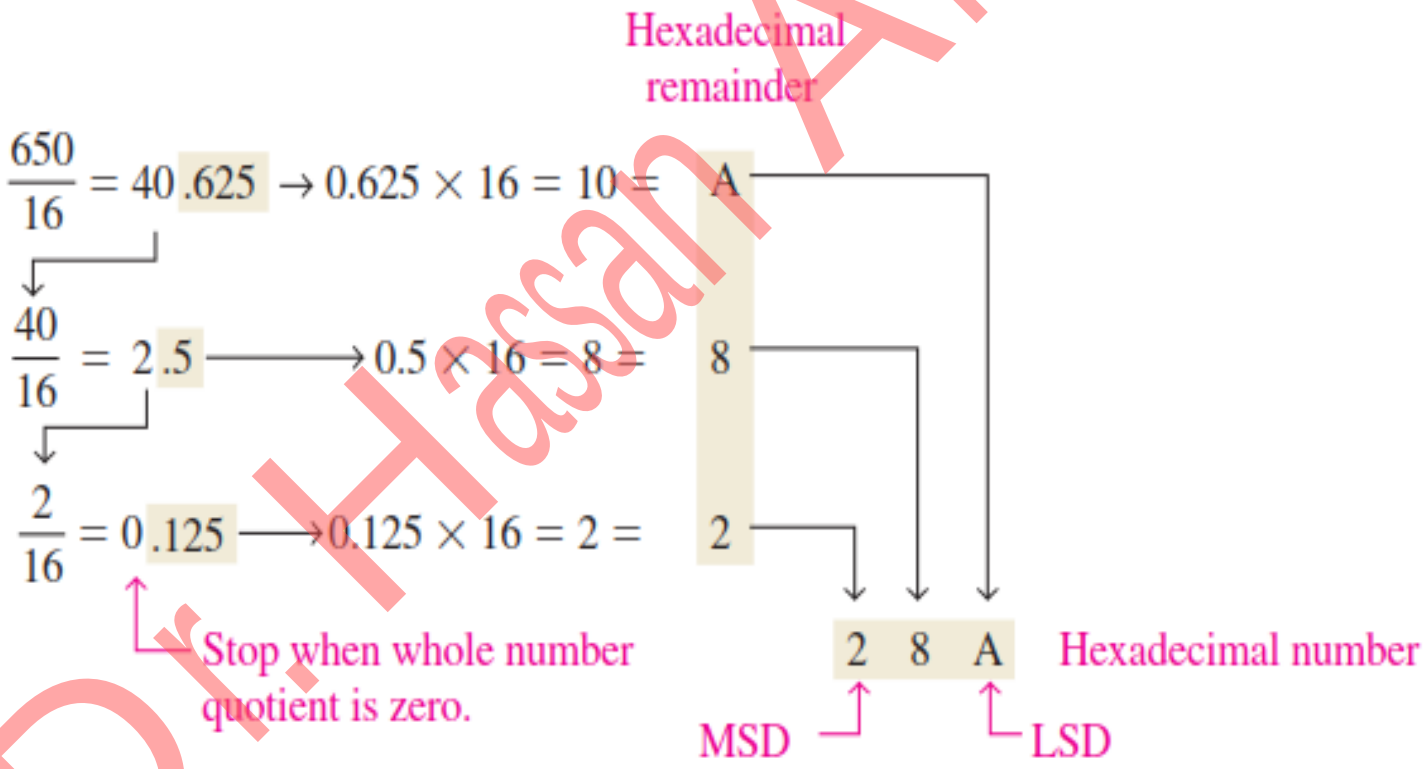
$1(4096) + 10(256) + 2(16) + 15(1) = 6703_{10}$

Decimal-to-Hexadecimal conversion

Example 3-17

Convert the decimal number 650 to hexadecimal by repeated division by 16.

Solution



Hexadecimal Addition

Addition can be done directly with hexadecimal numbers by remembering that the hexadecimal digits 0 through 9 are equivalent to decimal digits 0 through 9 and that hexadecimal digits A through F are equivalent to decimal numbers 10 through 15.

Example 3-18 Add the following hexadecimal numbers

(a) $23_{16} + 16_{16}$ (b) $58_{16} + 22_{16}$ (c) $2B_{16} + 84_{16}$ (d) $DF_{16} + AC_{16}$

Solution

(a)
$$\begin{array}{r} 23_{16} \\ +16_{16} \\ \hline 39_{16} \end{array}$$
right column: $3_{16} + 6_{16} = 3_{10} + 6_{10} = 9_{10} = 9_{16}$
left column: $2_{16} + 1_{16} = 2_{10} + 1_{10} = 3_{10} = 3_{16}$

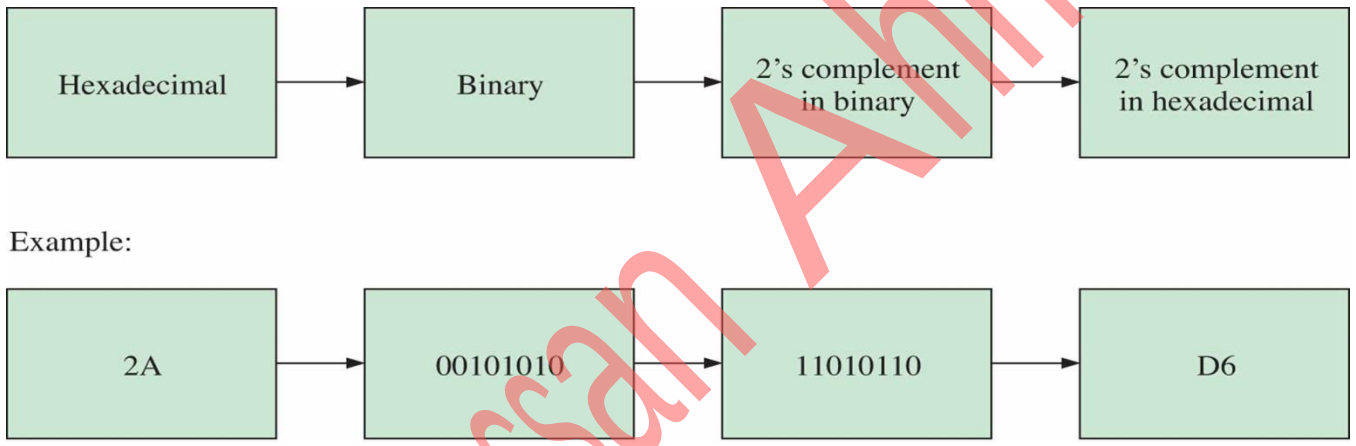
(b)
$$\begin{array}{r} 58_{16} \\ +\ 22_{16} \\ \hline 7A_{16} \end{array}$$
right column: $8_{16} + 2_{16} = 8_{10} + 2_{10} = 10_{10} = A_{16}$
left column: $5_{16} + 2_{16} = 5_{10} + 2_{10} = 7_{10} = 7_{16}$

(c)
$$\begin{array}{r} 2B_{16} \\ +\ 84_{16} \\ \hline AF_{16} \end{array}$$
right column: $B_{16} + 4_{16} = 11_{10} + 4_{10} = 15_{10} = F_{16}$
left column: $2_{16} + 8_{16} = 2_{10} + 8_{10} = 10_{10} = A_{16}$

(d)
$$\begin{array}{r} DF_{16} \\ +\ AC_{16} \\ \hline 18B_{16} \end{array}$$
right column: $F_{16} + C_{16} = 15_{10} + 12_{10} = 27_{10}$
$27_{10} - 16_{10} = 11_{10} = B_{16}$ with a 1 carry
left column: $D_{16} + A_{16} + 1_{16} = 13_{10} + 10_{10} + 1_{10} = 24_{10}$
$24_{10} - 16_{10} = 8_{10} = 8_{16}$ with a 1 carry

Hexadecimal Subtraction

- The 2's complement allows you to subtract by adding binary numbers.
- Since a hexadecimal number can be used to represent a binary number, it can be used to represent the 2's complement of binary number.



Example 3-19

Solution

$84_{16} - 2A_{16}$

$2A_{16} = 00101010$

2's complement of $2A_{16} = 11010110 = D6_{16}$

$$\begin{array}{rl} & 84_{16} \\ + & D6_{16} \\ \hline & \not{1}5A_{16} \end{array}$$

Add

Drop carry, as in 2's complement addition

Discard

The difference is $5A_{16}$.

4-2. Octal Numbers

- Octal uses eight numbers, which are 0 through 7, to represent numbers. There is no 8 or 9 character in octal.
- To count above 7, begin another column and start over:

  10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, ...

  - For instance, 15_8 in octal is equivalent to 13_{10} in decimal and D in hexadecimal.

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Octal-to-Decimal Conversion

Octal is also a weighted number system. The column weights are powers of 8, which increase from right to left.

Example 3-20

Express 3702_8 and 2374_8 in decimal.

Start by writing the column weights:

Solution

Weight: $8^3\ 8^2\ 8^1\ 8^0$

Octal number: 2 3 7 4

$$2374_8 = (2 \times 8^3) + (3 \times 8^2) + (7 \times 8^1) + (4 \times 8^0)$$
$$= (2 \times 512) + (3 \times 64) + (7 \times 8) + (4 \times 1)$$
$$= 1024 + 192 + 56 + 4 = 1276_{10}$$

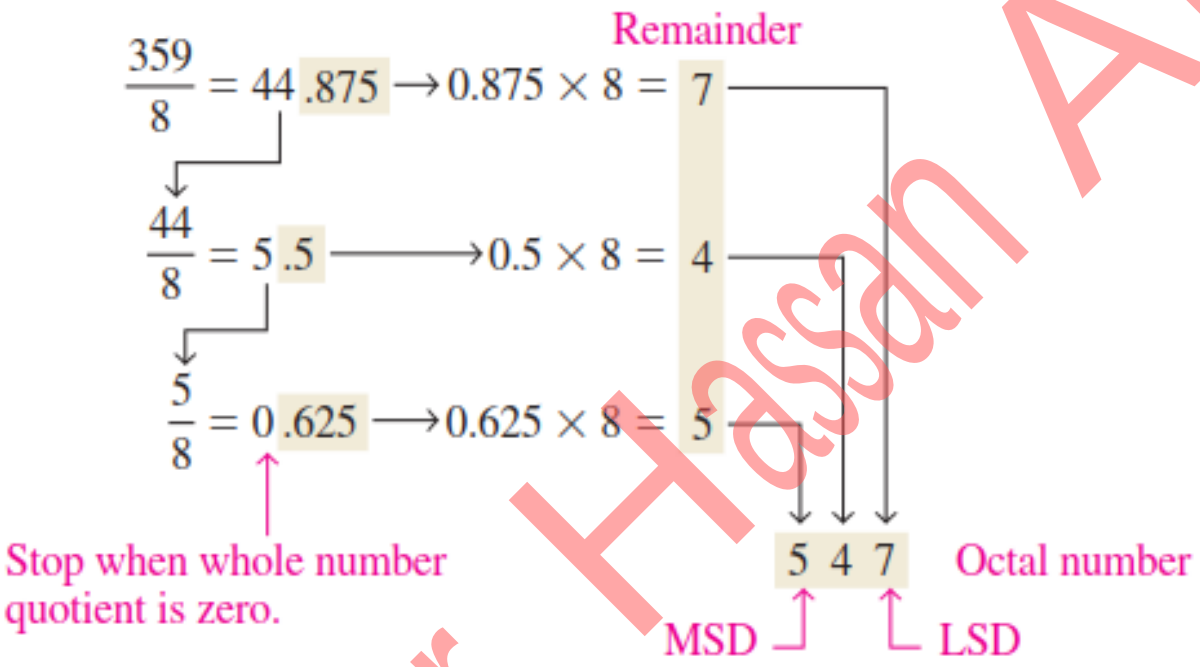
512 64 8 1

3 7 0 2_8

$$3(512) + 7(64) + 0(8) + 2(1) = 1986_{10}$$

Decimal-to-Octal Conversion

A method of converting a decimal number to an octal number is the repeated division-by-8 method.



Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Octal-to-Binary Conversion

To convert an octal number to a binary number, simply replace each octal digit with the appropriate three bits.

Octal Digit	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Example 3-21

Convert each of the following octal numbers to binary

(a) 13_8 (b) 25_8 (c) 140_8 (d) 7526_8

Solution

(a) 1 3 → 001011

(b) 2 5 → 010101

(c) 1 4 0 → 001100000

(d) 7 5 2 6 → 111101010110

Binary-to-Octal Conversion

Binary number can easily be converted to octal by grouping 3 bits at a time and writing the equivalent octal character for each group.

Decimal	Octal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	10	1000
9	11	1001
10	12	1010
11	13	1011
12	14	1100
13	15	1101
14	16	1110
15	17	1111

Example 3-22 Express $1\ 001\ 011\ 000\ 001\ 110_2$ in octal:

Solution Group the binary number by 3-bits starting from the right. Thus, 113016_8

(a) $\underbrace{110}_{6}\underbrace{101}_{5} = 65_8$

(b) $\underbrace{101}_{5}\underbrace{111}_{7}\underbrace{001}_{1} = 571_8$

(c) $\underbrace{100}_{4}\underbrace{110}_{6}\underbrace{011}_{3}\underbrace{010}_{2} = 4632_8$

(d) $\underbrace{011}_{3}\underbrace{010}_{2}\underbrace{000}_{0}\underbrace{100}_{4} = 3204_8$

4-3. Binary coded decimal (عشري مرمّز ثنائياً)

- Binary coded decimal (BCD) is a weighted code that is commonly used in digital systems when it is necessary to show decimal numbers such as in clock displays.
- The table illustrates the difference between straight binary and BCD.
  - BCD represents each decimal digit with a 4-bit code.
  - Notice that the codes 1010 through 1111 are not used in BCD.

Decimal	Binary	BCD
0	0000	0000
1	0001	0001
2	0010	0010
3	0011	0011
4	0100	0100
5	0101	0101
6	0110	0110
7	0111	0111
8	1000	1000
9	1001	1001
→ 10	1010	00010000
11	1011	00010001
12	1100	00010010
13	1101	00010011
14	1110	00010100
15	1111	00010101

The 8421 cod

The 8421 code is a type of BCD code. BCD means that each decimal digit, 0 through 9, is represented by a binary code of four bits.

The designation 8421 indicates the binary weights of the four bits (2^3, 2^2, 2^1, 2^0)

Decimal Digit	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

4-4. Digital codes (الرمازات الرقمية)

Gray code (رماز غراي)

- **Gray code** is an unweighted code that has a single bit change between one code word and the next in a sequence.
- Gray code is used to avoid problems in systems where an error can occur if more than one bit changes at a time.
- Like binary numbers, the Gray code can have any number of bits.
- Notice the single-bit change between successive (المتتالية/بالتسلسل) Gray code words.
  - **For instance**, in going from decimal **3** to decimal **4**, the Gray code changes from **0010** to **0110**, while the binary code changes from **0011** to **0100**, a change of three bits.

Decimal	Binary	Gray code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Binary-to-Gray code Conversation

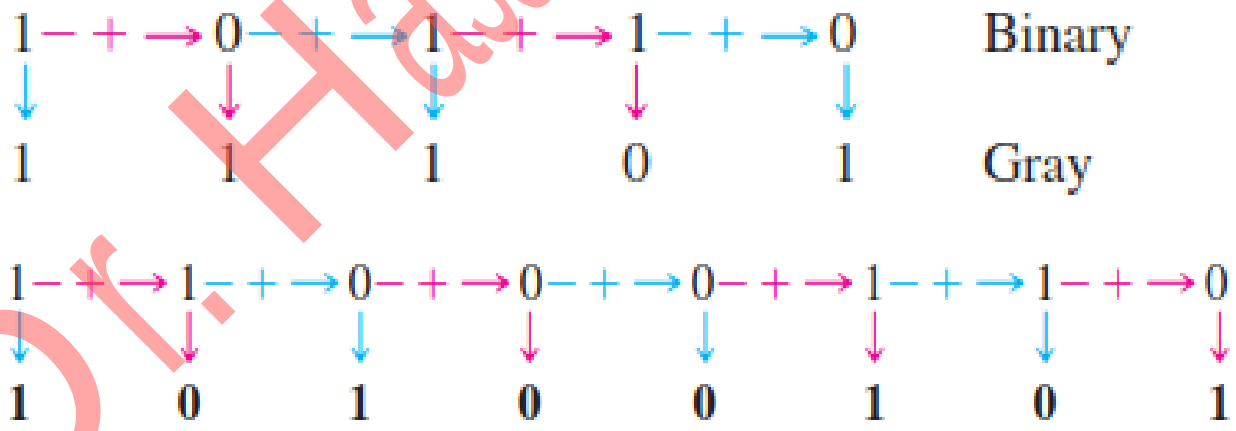
The following rules explain how to convert from a binary number to a Gray code word:

1. The most significant bit (MSB = left-most) in Gray code is the same as the corresponding MSB in the binary number.
2. Going from left to right, add each adjacent pair (زوج متجاور) of binary code to get the next Gray code bit.
3. Discard carries.

Example 3-24

Convert the binary numbers 10110 and 11000110 to Gray code.

Solution



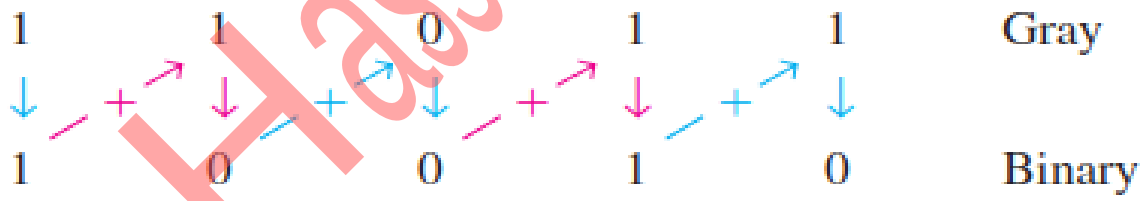
Gray-to-Binary code Conversation

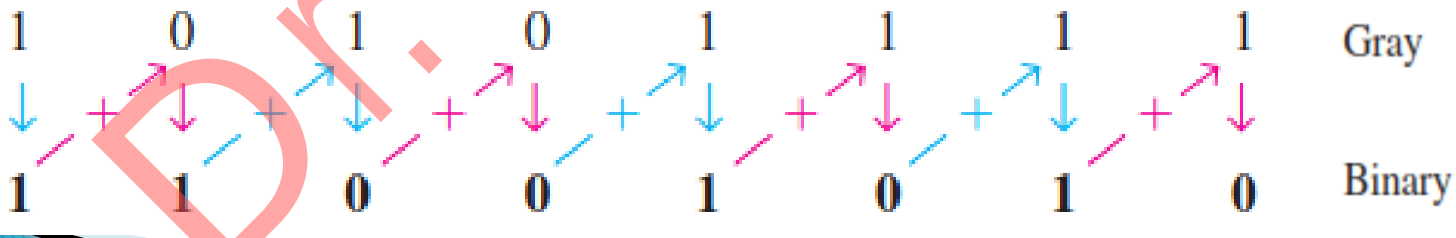
The following rules explain how to convert from a binary number to a Gray code word:

1. The most significant bit (MSB = left-most) in binary code is the same as the corresponding MSB in the Gray code.
2. Add each binary code bit generated to the Gray code bit in the next adjacent position (الموقع المجاور).
3. Discard carries.

Example 3-25 Convert the Gray codes 11011 and 10101111 to binary number.

Solution





Alphanumeric code

- The alphanumeric codes (رموز أبجدية عددية) are codes that represented numbers and alphabetic letters and symbols.
- At a minimum, an alphanumeric code must represent 10 decimal digits and 26 letters of alphabet, for a total of 36 items.
- This number requires six bits in each code combination because five bits are insufficient ($2^5 = 32$).
- The ASCII is the most common alphanumeric code.

ASCII: American Standard Code for Information Interchange

(الرماز الأمريكي القياسي لتبادل المعلومات)

- ASCII is a code for alphanumeric characters and control characters. In its original form, ASCII encoded 128 characters and symbols using 7-bits.
- The first 32 characters are control characters, that are based on obsolete teletype requirements, so these characters are generally assigned to other functions in modern usage.
- In 1981, IBM introduced extended ASCII, which is an 8-bit code and increased the character set to 256.
- Other extended sets (such as Unicode) have been introduced to handle characters in languages other than English (Asian).

American Standard Code for Information Interchange (ASCII).

Control Characters				Graphic Symbols											
Name	Dec	Binary	Hex	Symbol	Dec	Binary	Hex	Symbol	Dec	Binary	Hex	Symbol	Dec	Binary	Hex
NUL	0	0000000	00	space	32	0100000	20	@	64	1000000	40	`	96	1100000	60
SOH	1	0000001	01	!	33	0100001	21	A	65	1000001	41	a	97	1100001	61
STX	2	0000010	02	"	34	0100010	22	B	66	1000010	42	b	98	1100010	62
ETX	3	0000011	03	#	35	0100011	23	C	67	1000011	43	c	99	1100011	63
EOT	4	0000100	04	$	36	0100100	24	D	68	1000100	44	d	100	1100100	64
ENQ	5	0000101	05	%	37	0100101	25	E	69	1000101	45	e	101	1100101	65
ACK	6	0000110	06	&	38	0100110	26	F	70	1000110	46	f	102	1100110	66
BEL	7	0000111	07	'	39	0100111	27	G	71	1000111	47	g	103	1100111	67
BS	8	0001000	08	(	40	0101000	28	H	72	1001000	48	h	104	1101000	68
HT	9	0001001	09	)	41	0101001	29	I	73	1001001	49	i	105	1101001	69
LF	10	0001010	0A	*	42	0101010	2A	J	74	1001010	4A	j	106	1101010	6A
VT	11	0001011	0B	+	43	0101011	2B	K	75	1001011	4B	k	107	1101011	6B
FF	12	0001100	0C	,	44	0101100	2C	L	76	1001100	4C	l	108	1101100	6C
CR	13	0001101	0D	−	45	0101101	2D	M	77	1001101	4D	m	109	1101101	6D
SO	14	0001110	0E	.	46	0101110	2E	N	78	1001110	4E	n	110	1101110	6E
SI	15	0001111	0F	/	47	0101111	2F	O	79	1001111	4F	o	111	1101111	6F
DLE	16	0010000	10	0	48	0110000	30	P	80	1010000	50	p	112	1110000	70
DC1	17	0010001	11	1	49	0110001	31	Q	81	1010001	51	q	113	1110001	71
DC2	18	0010010	12	2	50	0110010	32	R	82	1010010	52	r	114	1110010	72
DC3	19	0010011	13	3	51	0110011	33	S	83	1010011	53	s	115	1110011	73
DC4	20	0010100	14	4	52	0110100	34	T	84	1010100	54	t	116	1110100	74
NAK	21	0010101	15	5	53	0110101	35	U	85	1010101	55	u	117	1110101	75
SYN	22	0010110	16	6	54	0110110	36	V	86	1010110	56	v	118	1110110	76
ETB	23	0010111	17	7	55	0110111	37	W	87	1010111	57	w	119	1110111	77
CAN	24	0011000	18	8	56	0111000	38	X	88	1011000	58	x	120	1111000	78
EM	25	0011001	19	9	57	0111001	39	Y	89	1011001	59	y	121	1111001	79
SUB	26	0011010	1A	:	58	0111010	3A	Z	90	1011010	5A	z	122	1111010	7A
ESC	27	0011011	1B	;	59	0111011	3B	[	91	1011011	5B	{	123	1111011	7B
FS	28	0011100	1C	<	60	0111100	3C	\	92	1011100	5C	|	124	1111100	7C
GS	29	0011101	1D	=	61	0111101	3D	]	93	1011101	5D	}	125	1111101	7D
RS	30	0011110	1E	>	62	0111110	3E	^	94	1011110	5E	~	126	1111110	7E
US	31	0011111	1F	?	63	0111111	3F	_	95	1011111	5F	Del	127	1111111	7F

Extended ASCII Characters

The extended ASCII characters are represented by an 8-bit code series from hexadecimal 80 to hexadecimal FF and can be grouped into the following general categories:

- Foreign (non-English) alphabetic characters (الأحرف/الرموز الأبجدية الأجنبية – غير الانكليزية),
- Foreign currency symbols (رموز العملات الأجنبية),
- Greek letters (الأحرف اليونانية),
- Mathematical symbols (الرموز الرياضية),
- Drawing characters (رموز الرسم),
- Bar graphing characters (رموز مخططات الأعمدة),
- and shading characters (رموز التظليل).

Unicode

- Unicode provides the ability to encode (ترميز) all of the characters used for the written languages of the world by assigning (تخصيص=تعيين) each character a unique numeric value and name utilizing (الاستخدام) the universal character set (UCS).
  - It is applicable (قابل للتطبيق) in computer applications dealing (تتعامل) with multi-lingual text, mathematical symbols, or other technical characters.
- Unicode consists of a number of related items (عناصر ذات صلة), such as:
  - ✓ character properties, (خصائص الرموز / الأحرف)
  - ✓ rules for text normalization, (قواعد تنضيد النص)
  - ✓ decomposition, collation, (التفكيك والترتيب = التصفيف)
  - ✓ bidirectional display order (ترتيب العرض ثنائي الاتجاه) (for the correct display of text containing both right-to-left scripts (مخطوطات), such as Arabic or Hebrew, and left-to-right scripts).

Selected Key Terms

Alphanumeric	Consisting of numerals, letters, and other characters.
ASCII	American Standard Code for Information Interchange; the most widely used alphanumeric code.
BCD	Binary coded decimal; a digital code in which each of the decimal digits, 0 through 9, is represented by a group of four bits.
Byte	A group of eight bits.
Hexadecimal	Describes a number system with a base of 16.
LSB	Least significant bit; the right-most bit in a binary whole number or code.
MSB	Most significant bit; the left-most bit in a binary whole number or code.
Octal	Describes a number system with a base of eight.

True/False Quiz

1. The octal number system is a weighted system with eight digits.
2. The binary number system is a weighted system with two digits.
3. MSB stands for most significant bit.
4. In hexadecimal, $9 + 1 = 10$.
5. The 1's complement of the binary number 1010 is 0101.
6. The 2's complement of the binary number 1111 is 0000.
7. The right-most bit in a signed binary number is the sign bit.
8. The hexadecimal number system has 16 characters, six of which are alphabetic characters.
9. BCD stands for binary coded decimal.
10. An error in a given code can be detected by verifying the parity bit.

1. T **2.** T **3.** T **4.** F **5.** T **6.** F **7.** F **8.** T **9.** T **10.** T

SELF-TEST

1. $3 \times 10^1 + 4 \times 10^0$ is
(a) 0.34 **(b)** 3.4 **(c)** 34 **(d)** 340

2. The decimal equivalent of 1000 is
(a) 2 **(b)** 4 **(c)** 6 **(d)** 8

3. The binary number 11011101 is equal to the decimal number
(a) 121 **(b)** 221 **(c)** 441 **(d)** 256

4. The decimal number 21 is equivalent to the binary number
(a) 10101 **(b)** 10001 **(c)** 10000 **(d)** 11111

5. The decimal number 250 is equivalent to the binary number
(a) 11111010 **(b)** 11110110 **(c)** 11111000 **(d)** 11111011

6. The sum of 1111 + 1111 in binary equals
(a) 0000 **(b)** 2222 **(c)** 11110 **(d)** 11111

7. The difference of 1000 − 100 equals
(a) 100 **(b)** 101 **(c)** 110 **(d)** 111

8. The 1's complement of 11110000 is
(a) 11111111 **(b)** 11111110 **(c)** 00001111 **(d)** 10000001

9. The 2's complement of 11001100 is
(a) 00110011 **(b)** 00110100 **(c)** 00110101 **(d)** 00110110

1. (c) **2.** (d) **3.** (b) **4.** (a) **5.** (a) **6.** (c) **7.** (a) **8.** (c) **9.** (b)

SELF-TEST

10. The decimal number +122 is expressed in the 2's complement form as
(a) 01111010 **(b)** 11111010 **(c)** 01000101 **(d)** 10000101

11. The decimal number −34 is expressed in the 2's complement form as
(a) 01011110 **(b)** 10100010 **(c)** 11011110 **(d)** 01011101

12. A single-precision floating-point binary number has a total of
(a) 8 bits **(b)** 16 bits **(c)** 24 bits **(d)** 32 bits

13. In the 2's complement form, the binary number 10010011 is equal to the decimal number
(a) −19 **(b)** +109 **(c)** +91 **(d)** −109

14. The binary number 10110011100101010100001 can be written in octal as
(a) 5471230_8 **(b)** 5471241_8 **(c)** 2634521_8 **(d)** 23162501_8

15. The binary number 10001101010001101111 can be written in hexadecimal as
(a) $AD467_{16}$ **(b)** $8C46F_{16}$ **(c)** $8D46F_{16}$ **(d)** $AE46F_{16}$

16. The binary number for $F7A9_{16}$ is
(a) 1111011110101001 **(b)** 1110111110101001
(c) 1111111010110001 **(d)** 1111011010101001

17. The BCD number for decimal 473 is
(a) 111011010 **(b)** 110001110011 **(c)** 010001110011 **(d)** 010011110011

10. (a) **11.** (c) **12.** (d) **13.** (d) **14.** (b) **15.** (c) **16.** (a) **17.** (c)

Problems & Solutions

Prob. 2-1

What is the weight of 6 in each of the following decimal numbers?
(a) 1386; (b) 54.692; (c) 671.920

Sol.

(a) $1386 = 1 \times 10^3 + 3 \times 10^2 + 8 \times 10^1 + 6 \times 10^0$
$= 1 \times 1000 + 3 \times 100 + 8 \times 10 + 6 \times 1$
The digit 6 has a weight of $10^0 = 1$

(b) $54{,}692 = 5 \times 10^4 + 4 \times 10^3 + 6 \times 10^2 + 9 \times 10^1 + 2 \times 10^0$
$= 5 \times 10{,}000 + 4 \times 1000 + 6 \times 100 + 9 \times 10 + 2 \times 1$
The digit 6 has a weight of $10^2 = 100$

(c) $671{,}920 = 6 \times 10^5 + 7 \times 10^4 + 1 \times 10^3 + 9 \times 10^2 + 2 \times 10^1 + 0 \times 10^0$
$= 6 \times 100{,}000 + 7 \times 10{,}000 + 1 \times 1000 + 9 \times 100 + 2 \times 10 + 0 \times 1$
The digit 6 has a weight of $10^5 = 100{,}000$

Prob. 2-2

Give the value of each digit in the following decimal numbers:
(a) 471; (b) 9356; (c) 125.000

Sol.

(a) $$471 = 4 \times 10^2 + 7 \times 10^1 + 1 \times 10^0$$
$$= 4 \times 100 + 7 \times 10 + 1 \times 1$$
$$= 400 + 70 + 1$$

(b) $$9{,}356 = 9 \times 10^3 + 3 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$$
$$= 9 \times 1000 + 3 \times 100 + 5 \times 10 + 6 \times 1$$
$$= 9{,}000 + 300 + 50 + 6$$

(c) $$125{,}000 = 1 \times 10^5 + 2 \times 10^4 + 5 \times 10^3$$
$$= 1 \times 100{,}000 + 2 \times 10{,}000 + 5 \times 1000$$
$$= 100{,}000 + 20{,}000 + 5{,}000$$

Prob. 2-3

Convert the following binary numbers to decimal:

(a) 11; (b) 100; (c) 111; (d) 1000;
(e) 1001; (f) 1100; (g) 1011; (h) 1111.

Sol.

(a) $11 = 1 \times 2^1 + 1 \times 2^0 = 2 + 1 = 3$
(b) $100 = 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 4$
(c) $111 = 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 4 + 2 + 1 = 7$
(d) $1000 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8$
(e) $1001 = 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 1 = 9$
(f) $1100 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8 + 4 = 12$
(g) $1011 = 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 8 + 2 + 1 = 11$
(h) $1111 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 8 + 4 + 2 + 1 = 15$

****

Prob. 2-4

Convert each binary number to decimal:

(a) 110011.11; (b) 101010.01; (c) 1000001.111; (d) 1111000.101;

Sol.

(a) $110011.11 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$
$= 32 + 16 + 2 + 1 + 0.5 + 0.25 = 51.75$

(b) $101010.01 = 1 \times 2^5 + 1 \times 2^3 + 1 \times 2^1 + 1 \times 2^{-2} = 32 + 8 + 2 + 0.25$
$= 42.25$

(c) $1000001.111 = 1 \times 2^6 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$
$= 64 + 1 + 0.5 + 0.25 + 0.125 = 65.875$

(d) $1111000.101 = 1 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^{-1} + 1 \times 2^{-3}$
$= 64 + 32 + 16 + 8 + 0.5 + 0.125 = 120.625$

Prob. 2-5 What is the highest decimal number that can be represented by each of the following numbers of binary digits (bits)?

(a) two; (b) three; (c) four; (d) five; (e) six; (f) seven; (g) eight; (h) nine; (i) ten; (j) eleven.

Sol.

(a) $2^2 - 1 = 3$
(b) $2^3 - 1 = 7$
(c) $2^4 - 1 = 15$
(d) $2^5 - 1 = 31$
(e) $2^6 - 1 = 63$
(f) $2^7 - 1 = 127$
(g) $2^8 - 1 = 255$
(h) $2^9 - 1 = 511$
(i) $2^{10} - 1 = 1023$
(j) $2^{11} - 1 = 2047$

****

Prob. 2-6 Convert each decimal number to binary using repeated division by 2:

(a) 15; (b) 21; (c) 28;

Sol.

(a)
$\frac{15}{2} = 7, R = 1$ (LSB)
$\frac{7}{2} = 3, R = 1$
$\frac{3}{2} = 1, R = 1$
$\frac{1}{2} = 0, R = 1$ (MSB)

(b)
$\frac{21}{2} = 10, R = 1$ (LSB)
$\frac{10}{2} = 5, R = 0$
$\frac{5}{2} = 2, R = 1$
$\frac{2}{2} = 1, R = 0$
$\frac{1}{2} = 0, R = 1$ (MSB)

(c)
$\frac{28}{2} = 14, R = 0$ (LSB)
$\frac{14}{2} = 7, R = 0$
$\frac{7}{2} = 3, R = 1$
$\frac{3}{2} = 1, R = 1$
$\frac{1}{2} = 0, R = 1$ (MSB)

Prob. 2-7

Add the binary numbers:

(a) 11 + 01 **(b)** 10 + 10 **(c)** 101 + 11
(d) 111 + 110 **(e)** 1001 + 101 **(f)** 1101 + 1011

Sol.

(a)
$$\begin{array}{r} 11 \\ +\,01 \\ \hline 100 \end{array}$$

(b)
$$\begin{array}{r} 10 \\ +\,10 \\ \hline 100 \end{array}$$

(c)
$$\begin{array}{r} 101 \\ +\,011 \\ \hline 1000 \end{array}$$

(d)
$$\begin{array}{r} 111 \\ +\,110 \\ \hline 1101 \end{array}$$

(e)
$$\begin{array}{r} 1001 \\ +\,0101 \\ \hline 1110 \end{array}$$

(f)
$$\begin{array}{r} 1101 \\ +\,1011 \\ \hline 11000 \end{array}$$

****

Prob. 2-8

Use direct subtraction on the following binary numbers:

(a) 11 − 1 **(b)** 101 − 100 **(c)** 110 − 101
(d) 1110 − 11 **(e)** 1100 − 1001 **(f)** 11010 − 10111

Sol.

(a)
$$\begin{array}{r} 11 \\ -\,01 \\ \hline 10 \end{array}$$

(b)
$$\begin{array}{r} 101 \\ -\,100 \\ \hline 001 \end{array}$$

(c)
$$\begin{array}{r} 110 \\ -\,101 \\ \hline 001 \end{array}$$

(d)
$$\begin{array}{r} 1110 \\ -\,0011 \\ \hline 1011 \end{array}$$

(e)
$$\begin{array}{r} 1100 \\ -\,1001 \\ \hline 0011 \end{array}$$

(f)
$$\begin{array}{r} 11010 \\ -\,10111 \\ \hline 00011 \end{array}$$

Prob. 2-9

Perform the following binary multiplications:

(a) 11×11	**(b)** 100×10	**(c)** 111×101
(d) 1001×110	**(e)** 1101×1101	**(f)** 1110×1101

Sol.

(a)
$$\begin{array}{r} 11 \\ \times\ 11 \\ \hline 11 \\ 11 \\ \hline 1001 \end{array}$$

(b)
$$\begin{array}{r} 100 \\ \times\ 10 \\ \hline 000 \\ 100 \\ \hline 1000 \end{array}$$

(c)
$$\begin{array}{r} 111 \\ \times\ 101 \\ \hline 111 \\ 000 \\ 111 \\ \hline 100011 \end{array}$$

(d)
$$\begin{array}{r} 1001 \\ \times\ 110 \\ \hline 0000 \\ 1001 \\ 1001 \\ \hline 110110 \end{array}$$

(e)
$$\begin{array}{r} 1101 \\ \times\ 1101 \\ \hline 1101 \\ 0000 \\ 1101 \\ 1101 \\ \hline 10101001 \end{array}$$

(f)
$$\begin{array}{r} 1110 \\ \times\ 1101 \\ \hline 1110 \\ 0000 \\ 1110 \\ 1110 \\ \hline 10110110 \end{array}$$

Prob. 2-10

Divide the binary numbers as indicated:

(a) $100 \div 10$ **(b)** $1001 \div 11$ **(c)** $1100 \div 100$

Sol.

(a) $\frac{100}{10} = 010$ (b) $\frac{1001}{0011} = 0011$ (c) $\frac{1100}{0100} = 0011$

****

Prob. 2-11

Determine the 1's complement of each binary number:

(a) 101	**(b)** 110	**(c)** 1010
(d) 11010111	**(e)** 1110101	**(f)** 00001

Sol.

(a) The 1's complement of 101 is 010.
(b) The 1's complement of 110 is 001.
(c) The 1's complement of 1010 is 0101.
(d) The 1's complement of 11010111 is 00101000.
(e) The 1's complement of 1110101 is 0001010.
(f) The 1's complement of 00001 is 11110.

Prob. 2-12

Determine the 2's complement of each binary number using either method:

(a) 10 **(b)** 111 **(c)** 1001 **(d)** 1101
(e) 11100 **(f)** 10011 **(g)** 10110000 **(h)** 00111101

Sol.

Take the 1's complement and add 1:

(a) 01 + 1 = 10
(b) 000 + 1 = 001
(c) 0110 + 1 = 0111
(d) 0010 + 1 = 0011
(e) 00011 + 1 = 00100
(f) 01100 + 1 = 01101
(g) 01001111 + 1 = 01010000
(h) 11000010 + 1 = 11000011

****

Prob. 2-13

Express each decimal number in binary as an 8-bit sign-magnitude number:

(a) +29 **(b)** −85 **(c)** +100 **(d)** −123

Sol.

(a) Magnitude of 29 = 0011101
+ 29 = 00011101

(b) Magnitude of 85 = 1010101
−85 = 11010101

(c) Magnitude of 100_{10} = 1100100
+100 = 01100100

(d) Magnitude of 123 = 1111011
−123 = 11111011

Prob. 2-14 Express each decimal number as an 8-bit number in the 1's complement form:

(a) −34 (b) +57 (c) −99 (d) +115

Sol.

(a) Magnitude of 34 = 0100010
−34 = 11011101

(b) Magnitude of 57 = 0111001
+57 = 00111001

(c) Magnitude of 99 = 1100011
−99 = 10011100

(d) Magnitude of 115 = 1110011
+115 = 01110011

****

Prob. 2-15 Express each decimal number as an 8-bit number in the 2's complement form:

(a) +12 (b) −68 (c) +101 (d) −125

Sol.

(a) Magnitude of 12 = 1100
+12 = 00001100

(b) Magnitude of 68 = 1000100
−68 = 10111100

(c) Magnitude of 101_{10} = 1100101
$+101_{10}$ = 01100101

(d) Magnitude of 125 = 1111101
−125 = 10000011

Prob. 2-16 Convert each pair of decimal numbers to binary and add using the 2's complement form: **(a)** 33 and 15 **(b)** 56 and −27 **(c)** −46 and 25 **(d)** −110 and −84

Sol.

(a)
$33 = 00100001$
$15 = 00001111$

$$\begin{array}{r} 00100001 \\ +\ 00001111 \\ \hline 00110000 \end{array}$$

(b)
$56 = 00111000$
$27 = 00011011$
$-27 = 11100101$

$$\begin{array}{r} 00111000 \\ +\ 11100101 \\ \hline 00011101 \end{array}$$

(c)
$46 = 00101110$
$-46 = 11010010$
$25 = 00011001$

$$\begin{array}{r} 11010010 \\ +\ 00011001 \\ \hline 11101011 \end{array}$$

(d)
$110_{10} = 01101110$
$-110_{10} = 10010010$
$84 = 01010100$
$-84 = 10101100$

$$\begin{array}{r} 10010010 \\ +\ 10101100 \\ \hline 100111110 \end{array}$$

****

Prob. 2-17 Perform each addition in the 2's complement form:

Sol.

(a) 00010110 + 00110011 **(b)** 01110000 + 10101111

(a)
$$\begin{array}{r} 00010110 \\ +\ 00110011 \\ \hline 01001001 \end{array}$$

(b)
$$\begin{array}{r} 01110000 \\ +\ 10101111 \\ \hline 100011111 \end{array}$$

Prob. 2-18

Perform each subtraction in the 2's complement form:

(a) 00110011 − 00010000 **(b)** 01100101 − 11101000

Sol.

(a)
$$\begin{array}{r} 00110011 \\ -\,00010000 \\ \hline \end{array} \qquad \begin{array}{r} 00110011 \\ +\,11110000 \\ \hline \not{1}\;00100011 \end{array}$$

(b)
$$\begin{array}{r} 01100101 \\ -\,11101000 \\ \hline \end{array} \qquad \begin{array}{r} 01100101 \\ +\,00011000 \\ \hline 01111101 \end{array}$$

****

Prob. 2-19

Convert each hexadecimal number to binary:

(a) 38_{16} **(b)** 59_{16} **(c)** $A14_{16}$ **(d)** $5C8_{16}$
(e) 4100_{16} **(f)** $FB17_{16}$ **(g)** $8A9D_{16}$

Sol.

(a) $38_{16} = 0011\ 1000$
(b) $59_{16} = 0101\ 1001$
(c) $A14_{16} = 1010\ 0001\ 0100$
(d) $5C8_{16} = 0101\ 1100\ 1000$
(e) $4100_{16} = 0100\ 0001\ 0000\ 0000$
(f) $FB17_{16} = 1111\ 1011\ 0001\ 0111$
(g) $8A9D_{16} = 1000\ 1010\ 1001\ 1101$

Prob. 2-19

Convert each binary number to hexadecimal:

(a) 1110	(b) 10	(c) 10111
(d) 10100110	(e) 1111110000	(f) 100110000010

Sol.

(a) $1110 = E_{16}$

(b) $10 = 2_{16}$

(c) $0001\ 0111 = 17_{16}$

(d) $1010\ 0110 = A6_{16}$

(e) $0011\ 1111\ 0000 = 3F0_{16}$

(f) $1001\ 1000\ 0010 = 982_{16}$

****

Prob. 2-20

Convert each hexadecimal number to decimal:

(a) 23_{16}	(b) 92_{16}	(c) $1A_{16}$	(d) $8D_{16}$
(e) $F3_{16}$	(f) EB_{16}	(g) $5C2_{16}$	(h) 700_{16}

Sol.

(a) $23_{16} = 2 \times 16^1 + 3 \times 16^0 = 32 + 3 = 35$

(b) $92_{16} = 9 \times 16^1 + 2 \times 16^0 = 144 + 2 = 146$

(c) $1A_{16} = 1 \times 16^1 + 10 \times 16^0 = 16 + 10 = 26$

(d) $8D_{16} = 8 \times 16^1 + 13 \times 16^0 = 128 + 13 = 141$

(e) $F3_{16} = 15 \times 16^1 + 3 \times 16^0 = 240 + 3 = 243$

(f) $EB_{16} = 14 \times 16^1 + 11 \times 16^0 = 224 + 11 = 235$

(g) $5C2_{16} = 5 \times 16^2 + 12 \times 16^1 + 2 \times 16^0 = 1280 + 192 + 2 = 1474$

(h) $700_{16} = 7 \times 16^2 = 1792$

Prob. 2-21

Convert each decimal number to hexadecimal:

(a) 8 (b) 14 (c) 33 (d) 52

Sol.

(a) $\frac{8}{16} = 0$, remainder $= 8$

hexadecimal number $= 8_{16}$

(b) $\frac{14}{16} = 0$, remainder $= 14 = E_{16}$

hexadecimal number $= E_{16}$

(c) $\frac{33}{16} = 2$, remainder $= 1$ (LSD)

$\frac{2}{16} = 0$, remainder $= 2$

hexadecimal number $= 21_{16}$

(d) $\frac{52}{16} = 3$, remainder $= 4$ (LSD)

$\frac{3}{16} = 0$, remainder $= 3$

hexadecimal number $= 34_{16}$

****

Prob. 2-22

Perform the following additions:

(a) $37_{16} + 29_{16}$ (b) $A0_{16} + 6B_{16}$ (c) $FF_{16} + BB_{16}$

Sol.

(a) $37_{16} + 29_{16} = 60_{16}$

(b) $A0_{16} + 6B_{16} = 10B_{16}$

(c) $FF_{16} + BB_{16} = 1BA_{16}$

Prob. 2-23

Perform the following subtractions:

(a) $51_{16} - 40_{16}$ (b) $C8_{16} - 3A_{16}$ (c) $FD_{16} - 88_{16}$

Sol.

(a) $51_{16} - 40_{16} = 11_{16}$
(b) $C8_{16} - 3A_{16} = 8E_{16}$
(c) $FD_{16} - 88_{16} = 75_{16}$

****

Prob. 2-24

Convert each octal number to decimal:

(a) 12_8 (b) 27_8 (c) 56_8 (d) 64_8

Sol.

(a) $12_8 = 1 \times 8^1 + 2 \times 8^0 = 8 + 2 = 10$
(b) $27_8 = 2 \times 8^1 + 7 \times 8^0 = 16 + 7 = 23$
(c) $56_8 = 5 \times 8^1 + 6 \times 8^0 = 40 + 6 = 46$
(d) $64_8 = 6 \times 8^1 + 4 \times 8^0 = 48 + 4 = 52$

Prob. 2-25 Convert each decimal number to octal by repeated division by 8:

(a) 15 (b) 27 (c) 46 (d) 70

Sol.

(a) $\frac{15}{8} = 1$, remainder = 7 (LSD)

$\frac{1}{8} = 0$, remainder = 1

octal number = 17_8

(b) $\frac{27}{8} = 3$, remainder = 3 (LSD)

$\frac{3}{8} = 0$, remainder = 3

octal number = 33_8

(c) $\frac{46}{8} = 5$, remainder = 6 (LSD)

$\frac{5}{8} = 0$, remainder = 5

octal number = 56_8

(d) $\frac{70}{8} = 8$, remainder = 6 (LSD)

$\frac{8}{8} = 1$, remainder = 0

$\frac{1}{8} = 0$, remainder = 1

octal number = 106_8

****

Prob. 2-26 Convert each octal number into binary:

(a) 13_8 (b) 57_8 (c) 101_8 (d) 321_8

Sol.

(a) $13_8 = 001\ 011$

(b) $57_8 = 101\ 111$

(c) $101_8 = 001\ 000\ 001$

(d) $321_8 = 011\ 010\ 001$

Prob. 2-27

Convert each binary number to Gray code:

(a) 11011 (b) 1001010 (c) 1111011101110

Sol.

(a) 1 + 1 + 0 + 1 + 1 *Binary*
1 0 1 1 0 *Gray*

(b) 1 + 0 + 0 + 1 + 0 + 1 + 0 *Binary*
1 1 0 1 1 1 1 *Gray*

(c) 1 + 1 + 1 + 1 + 0 + 1 + 1 + 1 + 0 + 1 + 1 + 1 + 0 *Binary*
1 0 0 0 1 1 0 0 1 1 0 0 1 *Gray*

****

Prob. 2-28

Convert each Gray code to binary:

(a) 1010 (b) 00010 (c) 11000010001

Sol.

(a) 1 0 1 0 *Gray*
1 1 0 0 *Binary*

(b) 0 0 0 1 0 *Gray*
0 0 0 1 1 *Binary*

(c) 1 1 0 0 0 0 1 0 0 0 1 *Gray*
1 0 0 0 0 0 1 1 1 1 0 *Binary*

The end of Lecture_04, chapter 2